Year 12 Examination, 2018

Question/Answer Booklet

MATHEMATICS SPECIALIST

Section Two: Calculator-assumed

Student Name/Number: ______________________________

Teacher Name: ______________________________

Time allowed for this section

Reading time before commencing work: ten minutes
Working time for this section: one hundred minutes

Materials required/recommended for this section

To be provided by the supervisor: This Question/Answer Booklet
Formula Sheet (retained from Section One)

To be provided by the candidate:
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in the WACE examinations

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	7	7	50	54	35
Section Two: Calculator-assumed	13	13	100	100	65
					100

Instructions to candidates

1. The rules for the conduct of School exams are detailed in the ____*School/College assessment policy*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer Booklet.

3. You must be careful to confine your responses to the specific questions asked and to follow any instructions that are specific to a particular question.

4. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

5. **Show all working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you **do not use pencil**, except in diagrams.

7. The Formula Sheet is **not** to be handed in with your Question/Answer Booklet.

Section Two: Calculator-assumed **65% (100 Marks)**

This section has **13** questions. Answer **all** questions. Write your answers in the spaces provided.

Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.

- Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
- Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

Suggested working time: 100 minutes.

Question 8 **(5 marks)**

Solve algebraically $|4-3x| = 5+|x+2|$.

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Question 9 **(6 marks)**

(a) Given that

$$\frac{1}{w} = \frac{1}{1-i} + \frac{2}{i} \ ,$$

express w in Cartesian form. (3 marks)

(b) Use your calculator to express i^i in Cartesian form, giving your answer correct to eight decimal places. (1 mark)

(c) Determine the smallest positive value β such that $i = \exp(i\beta)$. (1 mark)

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(d) Hence, or otherwise, determine the exact value found in part (b). (1 mark)

Question 10 **(6 marks)**

It is given that that $z = 2i$ is a solution of the quartic equation

$$z^4 - 2z^3 + mz^2 + nz + 104 = 0$$

in which m and n are real numbers.

(a) Determine the values of m and n. (2 marks)

(b) Factorise the quartic as a product of two quadratics and hence deduce the other three solutions of the quartic equation. (4 marks)

Question 11 **(9 marks)**

The planes $\mathcal{P}_1$ and $\mathcal{P}_2$ have Cartesian equations

$$2x + 3y + 2z = 3 \qquad \text{and} \qquad 3x + 5y + z = 5\ .$$

The planes $\mathcal{P}_1$ and $\mathcal{P}_2$ intersect in the line L.

(a) Evaluate the cross product $\boldsymbol{a} = (2\boldsymbol{i} + 3\boldsymbol{j} + 2\boldsymbol{k}) \times (3\boldsymbol{i} + 5\boldsymbol{j} + \boldsymbol{k})$. (1 mark)

(b) Use a geometrical argument to explain why $\boldsymbol{a}$ is parallel to L. (2 marks)

(c) Determine a vector equation for L given that the point (0,1,0) lies on this line. (2 marks)

(d) The plane $\mathcal{P}_3$ has the Cartesian equation

$$7x + 11y + az = b$$

for some constants a and b.

(i) What are the values of a and b if the planes $\mathcal{P}_1$, $\mathcal{P}_2$ and $\mathcal{P}_3$ have infinitely points in common? (3 marks)

(ii) What are the values of a and b if the planes $\mathcal{P}_1$, $\mathcal{P}_2$ and $\mathcal{P}_3$ have no point in common? (1 mark)

Question 12 **(7 marks)**

Given $f(x) = 2\cos x$ and $g(x) = \sqrt{1-x}$, determine:

(a) $f \circ g(x)$ and $g \circ f(x)$ (2 marks)

(b) the domain and range of $f \circ g$ (2 marks)

(c) the domain and range of $g \circ f$ (3 marks)

Question 13 **(5 marks)**

The curve $y = \sqrt{2}\cos x$ and the line $y = \frac{4}{\pi}x$ meet at $\left(\frac{\pi}{4}, 1\right)$.

(a) Determine the area bounded by the curve, the line and the y axis. (3 marks)

(b) Calculate the volume formed when this area is rotated about the x-axis. (2 marks)

Question 14 **(11 marks)**

A particle moves along a straight line. The displacement of the particle from the origin is x cm and its velocity is v cms^{-1}. The particle is moving such that $v^2 + 5x^2 = 25$.

(a) Determine the acceleration of the particle and hence show that the motion of the particle is simple harmonic with period $\frac{2\pi}{\sqrt{5}}$. (2 marks)

(b) Given the initial position of the particle was at the centre of oscillation, determine an expression for the displacement of the particle as a function of time. (2 marks)

(c) Find the maximum and minimum speed of the particle. (2 marks)

(d) Determine the distance travelled by the particle during the fourth second correct to two decimal places.

(3 marks)

(e) Is the particle travelling towards or away from the initial position at $t = 8$ seconds? Justify your answer.

(2 marks)

Question 15 **(4 marks)**

Use partial fractions to prove that

$$\int_2^4 \frac{(x+3)}{x(x-1)}\,dx = \ln\frac{81}{8}.$$

Question 16 **(4 marks)**

Consider the parabola P defined by $y^2 = 4x$ over the range $x \in [0,9]$.

(a) Show that the line L given by $y = ax$ meets the parabola at $x = 0$ and another point $x = b$ if $a > \frac{2}{3}$. (1 mark)

(b) Determine the value of a if the area contained between L and P over $0 \le x \le b$ equals the area contained between them for $b \le x \le 9$. (3 marks)

Question 17 **(15 marks)**

People living a certain city are susceptible to migraine attacks. The number of migraines suffered by each person in any given week is known to be normally distributed, with a mean 5.04 and a standard deviation of 1.8.

(a) What proportion of people suffer between 5 and 6 migraines per week? (2 marks)

(b) Calculate the upper quartile of the distribution. (2 marks)

(c) Researchers plan to test the effectiveness of acupuncture as treatment for migraine. They began by using acupuncture on a small random sample, and they found that for people in this sample the average number of migraines per week was 3.81, with a standard deviation of 1.3.

They now wish to use a larger sample to estimate the average number of migraines per week that would be suffered by people treated with acupuncture, to an accuracy of 0.2 at a 95% level of confidence.

How large does the sample need to be? Use the standard deviation from the preliminary trial to make your estimate. (2 marks)

(d) In a random sample of 200 people treated with acupuncture it is found that the average number of migraines is 4.55, with a standard deviation of 1.36.

Calculate a 95% confidence interval for μ_{acu}, the average number of migraine attacks suffered by people treated with acupuncture. (2 marks)

(e) Comment on the claim that 'the testing clearly shows that acupuncture reduces the frequency of migraine' attacks. (2 marks)

(f) Researchers also tested an alternative treatment for migraine attacks. They found that for the 100 hundred randomly selected people who were given a certain drug, the average number of migraine attacks was 4.18, with a standard deviation of 2.43.

Use a confidence interval to test the claim that 'these tests clearly show that the drug is more effective than acupuncture for treating migraine attacks'. (5 marks)

Question 18 **(9 marks)**

Two small rockets are launched simultaneously; rocket A from the origin of coordinates O, and the rocket B from a point P located 600 m the east of O.

The initial velocity of rocket A is given by $\boldsymbol{v_1}(0) = 40\boldsymbol{i} + 20\boldsymbol{j} + 80\boldsymbol{k}$, and throughout its flight its acceleration is $-8\boldsymbol{k}$.

The velocity of rocket B is the constant vector $v\cos\theta\,\boldsymbol{j} + v\sin\theta\,\boldsymbol{k}$, where the constant v is the speed and the constant θ is the angle of elevation.

You may assume that the unit vectors $\boldsymbol{i}, \boldsymbol{j}$ and $\boldsymbol{k}$ are aligned in the traditional manner, i.e. $\boldsymbol{i}$ points East and $\boldsymbol{j}$ points North in a horizontal plane $\mathcal{H}$, and $\boldsymbol{k}$ points vertically upwards. You may also assume that distances are measured in metres and time is measured in seconds.

(a) How far from the origin O is rocket A when it returns to the plane $\mathcal{H}$? (4 marks)

(b) Determine the value of θ if the flight paths of the two rockets meet. (3 marks)

(c) Determine the speed of rocket B if the rockets collide. (2 marks)

Question 19 **(7 marks)**

A spherical container of radius a is partly filled with water so that its depth at the deepest point is b .

The cross-section of the container is represented by the graph of $x^2 + y^2 = a^2$.

Show that the ratio of

the water in the partly filled container : water in the full sphere = $\dfrac{b^2(3a-b)}{4a^3} : 1$.

Does this fraction give the expected answers when $b = a$ and when $b = 2a$?

Question 20 **(12 marks)**

(a) Use the substitution $u = \ln\left(\frac{1000}{x}\right)$ to show that $\int \frac{1}{x\ln\left(\frac{1000}{x}\right)}\,dx = -\ln\left|\ln\left(\frac{1000}{x}\right)\right| + c$.

(3 marks)

(b) Consider the logistic differential equation for a population P, $\frac{dP}{dt} = rP(k-P)$, where r and k are constants and t is measured in years. Show that the population grows fastest when $P = \frac{k}{2}$.

(2 marks)

(c) Another model for population is given by the solution of the differential equation

$$\frac{dP}{dt} = cP\ln\left(\frac{k}{P}\right)$$

where c is a constant. Show that this population grows fastest when $P = \frac{k}{e}$. (2 marks)

(d) Given $k = 1000$, $c = 0.05$ and $r = 8\times10^{-5}$, and that $P = P_0 = 100$ when $t = 0$, compare the population sizes predicted by the two models when $t = 50$. (5 marks)

Additional working space

Question number: ___________________

Acknowledgements

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